Total No. of Printed Pages—7

HS/XII/A. Sc. Com/M/OC/21

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MATHEMATICS

(Old Course)

Full Marks : 100

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. Let $R = \{(a, b) : a, b \in N \text{ and } a + 3b = 12\}$. Find *(i)* dom($R$) and *(ii)* range($R$). 2

2. If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $(f \circ f)(x) = x$. 2

3. Find the matrix X such that $2A - B + X = 0$, where

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$$ 2

4. Find adj A, if $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$. 2

5. If $y = \sin^{-1}(\cos x) + \cos^{-1}(\sin x)$, then prove that

$$\frac{dy}{dx} + 2 = 0$$ 2

6. Evaluate : 2

$$\int x \log x dx$$

7. Form the differential equation of the family of curves given by $y = A\cos 2x + B\sin 2x$, where A and B are arbitrary constants. 2

8. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \cdots}}}\ \infty$, then prove that

$$\frac{dy}{dx} = \frac{\sin x}{(1 - 2y)}$$ 2

9. Find the value of K if the function

$$f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & \text{when } x \neq 0 \\ K, & \text{when } x = 0 \end{cases}$$

is continuous at $x = 0$. 2

10. Prove that

$$\int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x}\, dx = \frac{\pi}{4}$$ 2

11. Find the unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$. 2

12. Find the angle between the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$. 2

13. Prove that

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \frac{x}{2}$$ 2

14. Find the angle between the lines

$$\frac{x+1}{1} = \frac{4-y}{-1} = \frac{z-5}{2} \text{ and } \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$$ 2

15. If A and B are independent events such that $P(A) = 0 \cdot 3$ and $P(B) = 0 \cdot 4$, then find—

(i) $P(A \text{ and } B)$

(ii) $P(A \text{ or } B)$ 2

SECTION—B

16. Prove that

$$\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$$ 4

17. Using properties of determinant, prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$ 4

18. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in [2, 3]. 4

19. Evaluate : 4

$$\int \frac{xe^x}{(1+x)^2}\,dx$$

20. Evaluate : 4

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)}\,dx$$

21. Solve the differential equation : 4

$$(1+x^2)\frac{dy}{dx} + 2xy = \cos x$$

22. Evaluate $\int_0^2 (x^2 + x)dx$ as the limit of a sum. 4

23. Prove that

$$\int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$$ 4

Or

Evaluate : 4

$$\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$$

24. Find the equation of the plane through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and passing through the point $(1, 1, 1)$. 4

Or

Find the image of the point $(1, 6, 3)$ in the line

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$$ 4

25. Find the shortest distance between the lines

$$\vec{r} = (6\hat{i} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

and $$\vec{r} = (-9\hat{i} + \hat{j} - 10\hat{k}) + \mu(4\hat{i} + \hat{j} + 6\hat{k})$$ 4

SECTION—C

26. Solve the following system of equations using matrix method : 6

$$2x - 3y + 5z = 16$$
$$3x + 2y - 4z = -4$$
$$x + y - 2z = -3$$

27. Using integration, find the area of ΔABC, whose vertices are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$. 6

28. In a bolt factory, three machines *A*, *B* and *C* manufacture 25%, 35% and 40% of the total production respectively. Of their respective outputs, 5%, 4% and 2% are defective. A bolt is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by the machine *C*. 6

29. A square piece of tin of side 18 cm is to be made into a box without the top, but cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum? Also, find the maximum volume of the box. 6

Or

Show that the maximum volume of the cylinder which can be inscribed in a sphere of radius $5\sqrt{3}$ cm is (500π) cm^3. 6

30. A manufacturer produces two types of steel trunk. He has two machines, *A* and *B*. The first type of trunk requires 3 hours on machine *A* and 3 hours on machine *B*. The second type of trunk requires 3 hours on machine *A* and 2 hours on machine *B*. Machines *A* and *B* can work at most for 18 hours and 15 hours per day respectively. He earns a profit of ₹ 30 and ₹ 25 per trunk of the first type and second type resepctively. How many trunks of each type must he make each day to make the maximum profit? 6

Or

If a young man rides his motorcycle at 25 km per hour, he has to spend ₹ 2 per km on petrol. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹ 5 per km. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it. 6

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