Total No. of Printed Pages—12

HS/XII/A. Sc. Com/M/22

2 0 2 2

MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 3 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. If $A = \{1, 2, 3, \cdots, 13, 14\}$ and R is a relation on A given by $R = \{(x, y) : 3x - y = 0\}$. Find the range of R. 1

Or

Find the principal value of $\tan^{-1}(-1)$. 1

2. Find $g \circ f$, if $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$. 1

3. Construct a 2×2 matrix whose elements are given by

$$a_{ij} = \frac{1}{2}|-3i + j|$$ 1

Or

Find AB, if $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ 1

4. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$. 1

Or

Find the scalar and vector components of the vector with initial point $A(2, 1)$ and terminal point $B(-5, 7)$. 1

5. Prove that $y = e^x + 1$ is a solution of the differential equation $y'' - y' = 0$. 1

6. If $y = \sin(ax + b)$, then find $\frac{dy}{dx}$. 1

Or

Prove that logarithmic function, i.e., $f(x) = \log x$ is strictly increasing on $(0, \infty)$. 1

7. Evaluate : 1

$$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$$

8. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at $x = 2$. 1

9. Evaluate : 1

$$\int \sec x(\sec x + \tan x)dx$$

Or

Evaluate : 1

$$\int_2^3 \frac{1}{x}dx$$

10. Find the Cartesian equations of the plane $\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2$. 1

11. Given that E and F are events such that $P(E)=0\cdot 6$, $P(F)=0\cdot 3$ and $P(E\cap F)=0\cdot 2$. Find $P(E/F)$. 1

12. Given two independent events A and B such that $P(A)=0\cdot 3$ and $P(B)=0\cdot 6$, find P(neither A nor B). 1

13. Find $\dfrac{dy}{dx}$ if $(x-y)=\pi$. 1

Or

Find $\dfrac{dx}{d\theta}$ if $x=a(\theta-\sin\theta)$. 1

14. Find the projection of the vector $\vec{a}=\hat{i}+3\hat{j}+7\hat{k}$ on the vector $\vec{b}=7\hat{i}-\hat{j}+8\hat{k}$. 1

Or

Show that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$. 1

15. Find the rate of change of the area of a circle with respect to its radius r at $r=6$. 1

16. Write the degree and order of the following differential equation : 1

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2y = 0$$

Choose the correct answer :

17. The maximum value of $Z = 3x + 4y$

subject to the constraints

$$x + y \le 4$$
$$x \ge 0,\ y \ge 0 \quad \text{is}$$

(a) 12

(b) 16

(c) 28

(d) 18 1

18. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to

(a) $\frac{7\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$ 1

Or

If $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = (3 - x^3)^{1/3}$, then $f \circ f(x)$ is

(a) $x^{1/3}$

(b) x^3

(c) x

(d) None of the above 1

19. The derivative of $\cos(\sqrt{x})$ is

(a) $\sin(\sqrt{x})$

(b) $\dfrac{\sin\sqrt{x}}{2\sqrt{x}}$

(c) $-\dfrac{\cos(\sqrt{x})}{2\sqrt{x}}$

(d) $-\dfrac{\sin(\sqrt{x})}{2\sqrt{x}}$ 1

20. $\displaystyle\int \frac{dx}{\sin^2 x \cos^2 x}$ is equal to

(a) $\tan x + \cot x + c$

(b) $\tan x - \cot x + c$

(c) $\tan x \cdot \cot x + c$

(d) None of the above 1

Or

$\int x^2 e^{x^3} dx$ is equal to

(a) $\frac{1}{3}e^{x^3}+c$

(b) $\frac{1}{3}e^{x^2}+c$

(c) $\frac{1}{2}e^{x^3}+c$

(d) $\frac{1}{2}e^{x^2}+c$ 1

SECTION—B

21. Let $f: N \to N$ be defined by

$$f(n)=\begin{cases}\frac{n+1}{2}, & \text{if } n \text{ is odd}\\ \frac{n}{2}, & \text{if } n \text{ is even for all } n \in N\end{cases}$$

State whether the function f is bijective. Justify your answer. 2

Or

Write the following function in simplest form : 2

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$$

22. If

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

then verify that $A'A = I$, I is the identity matrix. 2

Or

Find the value of x if

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$$ 2

23. Find the second-order derivative of $x^3 \log x$. 2

24. Show that

$$f(x) = \begin{cases} x^3 - 3 & \text{if } x \le 2 \\ x^2 + 1 & \text{if } x > 2 \end{cases}$$

is continuous at $x = 2$. 2

Or

Prove that the function f is given by $f(x) = |x - 1|$, $x \in \mathbb{R}$ is not differentiable at $x = 1$. 2

25. Evaluate : 2

$$\int x \log x \, dx$$

Or

Evaluate : 2

$$\int \frac{3x^2}{x^6 + 1} dx$$

26. Relation R in the set Z of all integers defined as $R = \{(x, y) : x - y \text{ is an integer}\}$. Prove that R is reflexive as well as symmetric. 2

Or

Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x$ is both one-one and onto. 2

SECTION—C

27. Find $A^2 - 5A + 6I$, if

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$ 4

Or

Using properties of determinant, prove that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$ 4

28. Solve the following graphically : 4

Minimize $Z = 3x + 5y$

subject to the constraints

$$x + 3y \geq 3$$
$$x + y \geq 2$$
$$x,\ y \geq 0$$

29. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. 4

Or

Using differentials, find the approximate value of $(26)^{1/3}$. 4

30. Solve the following homogeneous differential equation : 4

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

Or

Find the general solution of the following linear differential equation : 4

$$(1 + x^2)dy + 2xy\, dx = \cot x\, dx \ (x \neq 0)$$

31. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $(\vec{a} + \lambda\vec{b})$ is perpendicular to $\vec{c}$, then find the value of λ. 4

Or

Find the vector and Cartesian equations of the line that passes through the points (3, − 2, − 5) and (3, − 2, 6). 4

32. Using properties of integral calculus, prove that

$$\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx = \frac{\pi}{4}$$ 4

Or

Evaluate $\int_2^3 x^2 dx$ as a limit of sum. 4

SECTION—D

33. Solve the following system of linear equations using matrix method : 6

$$\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$$

Or

Using elementary transformation, find the inverse of the following matrix : 6

$$A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

34. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to $x - y + z = 0$. 6

35. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible? 6

Or

Using integration, find the area of region bounded by the triangle whose vertices are $(-1, 0)$, $(1, 3)$ and $(3, 2)$. 6

36. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two-headed coin? 6

Or

A die is thrown 6 times. If 'getting an odd number' is a success, then what is the probability of—

(i) 5 successes;

(ii) at least 5 successes;

(iii) at most 5 successes? 6

★★★