Time: 9:00 AM to 10:15 AM

Question Paper Code: 61

Roll No. of Student's														

Write the question paper code mentioned above on YOUR OMR Answer Sheet (in the space provided), otherwise your Answer Sheet will NOT be evaluated. Note that the same Question Paper Code appears on each page of the question paper.

Instructions to Candidates:

1. Use of mobile phone, smart watch, and iPad during examination is STRICTLY **PROHIBITED.**
2. In addition to this question paper, you are given OMR Answer Sheet along with candidate's copy.
3. On the OMR sheet, make all the entries carefully in the space provided **ONLY** in **BLOCK CAPITALS** as well as by properly darkening the appropriate bubbles.
   Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.
4. On the OMR Answer Sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling the bubbles.
5. Your **14-digit roll number and date of birth** entered on the OMR Answer Sheet shall remain your login credentials means login id and password respectively for accessing your performance / result in Indian Olympiad Qualifier in Physics 2021-22 (Part I).
6. Question paper has two parts. In part A1 (Q. No.1 to 24) each question has four alternatives, out of which **only one** is correct. Choose the correct alternative and fill the appropriate bubble, as below.

   Q.No.12 a c d

   In part A-2 (Q. No. 25 to 32) each question has four alternatives out of which any number of alternative(s) (1, 2, 3, or 4) may be correct. You have to choose **all** correct alternative(s) and fill the appropriate bubble(s), as shown

   Q.No.30 a c
7. For **Part A-1,** each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In **Part A-2,** you get 6 marks if all the correct alternatives are marked and no incorrect. No negative marks in this part.
8. Rough work should be done in the space provided. There are **11** printed pages in this paper
9. Use of **non- programmable scientific** calculator is allowed.
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting answer paper, take away the question paper & Candidate's copy of OMR for your reference.

Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR answer sheet.

OMR answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED. Scratching or overwriting may result in a wrong score.

DO NOT WRITE ON THE BACK SIDE OF THE OMR ANSWER SHEET.

Instructions to Candidates (Continued) :
You may read the following instructions after submitting the answer sheet.

12. **Comments/Inquiries/Grievances regarding this question paper, if any, can be shared on the Inquiry/Grievance column on www.iapt.org.in on the specified format till January 22, 2022.**
13. **The answers/solutions to this question paper will be available on the website: www.iapt.org.in by January 20, 2022.**

14. **CERTIFICATES and AWARDS:**
Following certificates are awarded by IAPT to students, successful in the Indian Olympiad Qualifier in Physics 2021-22 (Part I)
(i) "CENTRE TOP 10 %" To be downloaded from iapt.org.in after 15.03.22
(ii) "STATE TOP 1 %" Will be dispatched to the examinee
(iii) "NATIONAL TOP 1 %" Will be dispatched to the examinee
(iv) "GOLD MEDAL & MERIT CERTIFICATE" to all students who attend OCSC – 2022 at HBCSE Mumbai
Certificate for centre toppers shall be uploaded on iapt.org.in
15. List of students (with centre number and roll number only) having score above MAS will be displayed on the website: **www.iapt.org.in** by **February 06, 2022 See the Minimum Admissible Score Clause** on the student's brochure on the web.
16. List of students eligible for evaluation of IOQP 2021-22 (Part II) shall be displayed on www.iapt.org.in by February 10, 2022.

Physical constants you may need….

Mass of electron $m_e = 9.10\times10^{-31} kg$	Magnitude of charge on electron $e = 1.60\times10^{-19} C$
Mass of proton $m_p = 1.67\times10^{-27} kg$	Permittivity of free space $\varepsilon_0 = 8.85\times10^{-12} C^2N^{-1}m^{-2}$
Acceleration due to gravity g = 9.81 ms^{-2}	Permeability of free space $\mu_0 = 4\pi\times10^{-7}\ Hm^{-1}$
Universal gravitational constant $G = 6.67\times10^{-11} Nm^2Kg^{-2}$	Planck's constant $h = 6.63\times10^{-34} Js$
Universal gas constant $R = 8.31\ Jmol^{-1} K^{-1}$	Faraday constant $= 96,500\ Cmol^{-1}$
Boltzmann constant $k = 1.38\times10^{-23} J\,K^{-1}$	Rydberg constant $R = 1.097\times10^{7} m^{-1}$
Stefan's constant $\sigma = 5.67\times10^{-8} Wm^{-2}K^{-4}$	Density of water at $4^{\circ}C$, $\rho = 1.0\times10^{3} kg\,m^{-3}$
Avogadro's constant $A = 6.023\times10^{23} mol^{-1}$	Density of mercury $\rho = 13.6\times10^{3}\ kg\,m^{-3}$
Speed of light in free space $c = 3.0\times10^{8} ms^{-1}$	Speed of sound in air = 330 ms^{-1}

PHYSICS 2021-22 (Part I) (NSEP 2021 – 22)

Time: 75 Minute **Max. Marks: 120**

Attempt All Thirty Two Questions

A – 1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBBLE THE CORRECT OPTION.

1. Consider the process of the melting of a spherical ball of ice originally at 0° Assuming that the heat is being absorbed uniformly through the surface and the rate of absorption is proportional to the instantaneous surface area. Which of the following is true for the radius (r) of the ice ball at any instant of time? Assume that the initial radius of the ice ball at $t = 0$ *is* $r = R_0$ and that the shape of the ball always remains spherical during melting. Also assume that L and ρ are respectively the latent heat and density of ice at 0°

(a) radius decreases exponentially with time as $r = R_0 e^{-\frac{kt}{\rho L}}$. Here k is constant

(b) radius decreases exponentially with time as $r = R_0 e^{-\frac{k\rho t}{2L}}$

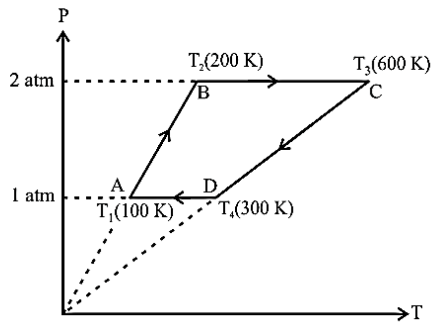
(c) radius of the ice ball decreases with time linearly with a slope $-\frac{k}{\rho L}$

(d) radius of the ice ball decreases with time linearly with a slope $-\frac{k\rho}{2L}$

2. The work done by the three moles of an ideal gas in the cyclic process ABCD shown in the diagram is approximately. Given that

$T_1 = 100\,K$, $T_2 = 200\,K$ and
$T_3 = 600K$, $T_4 = 300\,K$

(a) 7.5 kJ (b) 5.0 kJ
(c) 2.5 kJ (d) Zero



3. The molar specific heat capacity of a certain gas is expressed as $C = C_V + \alpha \frac{P}{T}$.

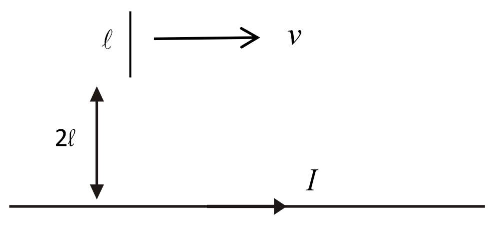
The equation of state for the process can be written as (α & A are constant)

(a) PV = RT (b) $V = \alpha T^2$ (c) $V^2 = \alpha \ln T$ (d) $T = Ae^{\frac{V}{\alpha}}$

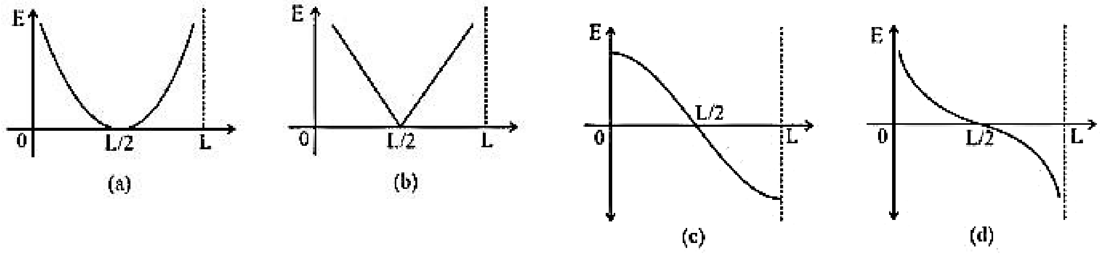
4. A metal bar of length ℓ moves with a velocity v parallel to an infinitely long straight wire carrying a current I as shown in the figure. If the nearest end of the perpendicular bar always remains at a distance 2ℓ from the current carrying wire, the potential difference (in volt) between two ends of the moving bar is

(a) $\frac{\mu_0 Iv}{2\pi}$ (b) $\frac{\mu_0 Iv}{6\pi}$

(c) $\frac{\mu_0 Iv}{2\pi}\ell$ (d) $\frac{\mu_0 Iv}{2\pi}\ell$



5. Two point charges +Q each are located at (0, 0) and (L, 0) at a distance L apart on the X - axis. The electric field (E) in the region $0 < x < L$ is best represented by



(a) Fig. a (b) Fig. b (c) Fig. c (d) Fig. d

6. A long straight wire AB of length L (L >> a, L >> b) and resistance R is connected to a time varying source of emf V(t). The variation of applied emf V(t) with time is shown in Fig. B. A circular metallic loop of radius r = b is placed coplanar with the current carrying wire with its centre at a distance 'a' from the axis of the wire as shown. The induced current in the loop is

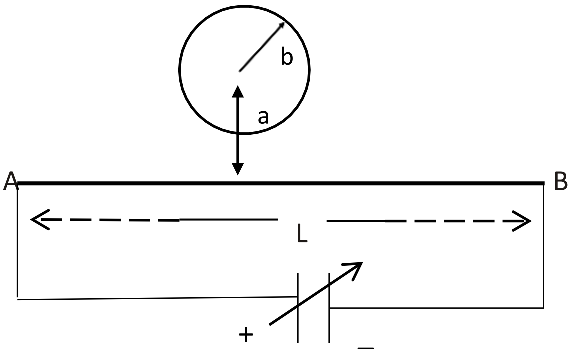


Fig. A V(t)

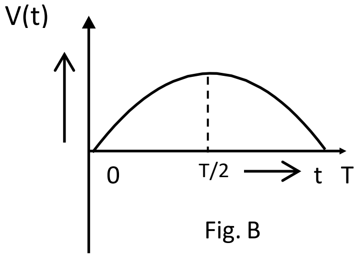


Fig. B

(a) clockwise from 0 to T/2 and anticlockwise from T/2 to T
(b) anticlockwise from 0 to T/2 and clockwise from T/2 to T
(c) clock wise from 0 to T
(d) anticlockwise from 0 to T

7. A simple circuit consists of a known resistance $R_A = 2\ M\Omega$ and an unknown resistance R_B both in series with a battery of 9 volt and negligible internal resistance. When the voltmeter is connected across the resistance R_A, it measures 3 volt but when the same voltmeter is connected across R_B it reads 4.5 volt. The voltmeter measures 9 V across the battery. Considering that the voltmeter has a finite resistance r, the correct option is

(a) $R_B = 3M\Omega$ *and* $r = 6.0M\Omega$ (b) $R_B = 2.5M\Omega$ *and* $r = 6.0M\Omega$

(c) $R_B = 4M\Omega$ *and* $r = 12M\Omega$ (d) $R_B = 4.5M\Omega$ *and* $r = 6.0M\Omega$

8. The optical powers of the objective and the eyepiece of a compound microscope are 100 D and 20 D respectively. The microscope magnification being equal to 50 when the final image is formed at d = 25 cm i.e., the least distance of distinct vision. If the separation between the objective and the eyepiece is increased by 2 cm, the magnification of the microscope will be

(a) 62 (b) 50 (c) 38 (d) 25

9. A hollow non-conducting cone of base radius R = 50 cm and semi vertical angle of 15^0 has been uniformly charged on its curved surface up to three-fourth of its slant length from base with a surface charge density $\sigma = 2.5\,\mu C/m^2$. The electric field produced at the location of the vertex of the cone is

(a) $\dfrac{\sigma \ln 2}{2\varepsilon_0}$ (b) $\dfrac{\sigma \ln 2}{4\varepsilon_0}$ (c) $\dfrac{\sigma \ln 2}{8\varepsilon_0}$ (d) $\dfrac{\sigma \ln 2}{16\varepsilon_0}$

10. A freely falling spherical rain drop gathers moisture (maintaining its spherical shape all the way) from the atmosphere at a rate $\dfrac{dm}{dt} = kt^2$ where t is the time and m is the instantaneous mass of the drop, the constant $k = 12\,gm/s^3$. If the drop, of initial mass $m_0 = 2gm$, starts falling from rest, the instantaneous velocity of the drop exactly after 5 second shall be (ignore air friction and air buoyancy)

(a) 12.4 ms^{-1} (b) 49.0 ms^{-1} (c) 122.5 ms^{-1} (d) data insufficient

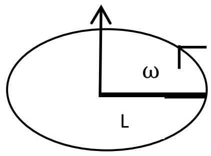
11. Two planets, each of mass M and radius R are positioned (at rest) in space, with their centres a distance 4R apart. You wish to fire a projectile from the surface of one planet to the other. The minimum initial speed for which this may be possible is

(a) $\sqrt{\dfrac{2GM}{5R}}$ (b) $\sqrt{\dfrac{2GM}{3R}}$ (c) $\sqrt{\dfrac{4GM}{3R}}$ (d) $\sqrt{\dfrac{3GM}{2R}}$

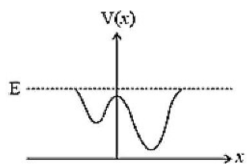
12. A thin uniform metallic rod of length L and radius R rotates with an angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. The density and the Young's modulus of the material of the rod are ρ and Y respectively.
The elongation in its length is

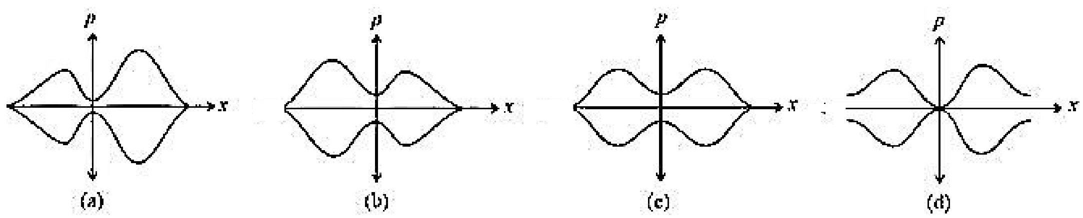
(a) $\dfrac{\rho\omega^2 L^3}{6Y}$ (b) $\dfrac{\rho\,\omega^2 L^3}{3Y}$

(c) $\dfrac{\rho\,\omega^2 R L^2}{2Y}$ (d) $\dfrac{\rho\,\omega^2 L^3}{2Y}$



13. Consider a particle of mass m with a total energy E moving in a one dimensional potential field. The potential V(x) is plotted against x in the figure beside.
The plot of momentum – position graph of this particle is qualitatively best represented by





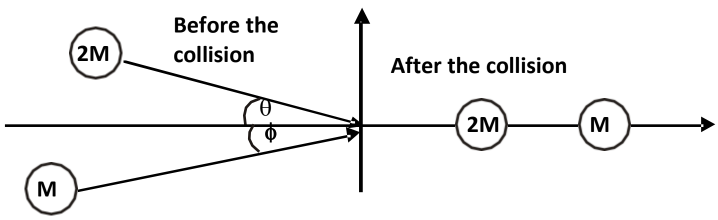
All plots are symmetrical about x - axis

(a) Fig. a (b) Fig. b (c) Fig. c (d) Fig. d

14. Knowing that the parallel currents attract, the inward pressure on the curved surface of a thin walled, long hollow metallic cylinder of radius R = 50 cm carrying a current of i = 2 amp parallel to its axis distributed uniformly over the entire circumference, is

(a) $2.05\times10^{-1}\,Nm^{-2}$ (b) $2.55\times10^{-3}\,Nm^{-2}$ (c) $2.05\times10^{-5}\,Nm^{-2}$ (d) $2.55\times10^{-7}\,Nm^{-2}$

15. Two masses move on a collision path as shown. Before the collision the object with mass 2M moves with a speed v making an angle $\theta=\sin^{-1}\frac{3}{5}$ to the x-axis while the object with mass M moves with a speed $\frac{3}{2}v$ making an angle $\phi=\sin^{-1}\frac{4}{5}$ with the x-axis. After the collision the object of mass 2M is observed to be moving to the right along the x-axis with a speed of $\frac{4}{5}v$. There are no external forces acting during the collision. The correct option is



(a) The velocity of mass M, after the collision, is zero.
(b) The centre of mass is moving along x-axis before the collision.
(c) The velocity of centre of mass after the collision is $\frac{5}{2}v$
(d) The total linear momentum of the system before the collision along x - axis is $\frac{5}{6}Mv$

16. A large hemispherical water tank of radius R is filled with water initially upto a height $h=\frac{R}{2}$. The water starts dripping out through a small orifice of cross section area 'a' at its spherical bottom. The time taken to get the tank completely empty (neglect viscosity) is

(a) $t=\frac{19\pi R^2}{60a}\sqrt{\frac{R}{g}}$ (b) $t=\frac{3\pi R^2}{10a}\sqrt{\frac{R}{g}}$

(c) $t=\frac{17\pi R^2}{60a}\sqrt{\frac{R}{g}}$ (d) $t=\frac{\pi R^2}{4a}\sqrt{\frac{R}{g}}$

17. If Pascal (Pa), the unit of pressure volt (V), the unit of potential and meter (L), the unit of length are taken as fundamental units, the dimensional formula for the permittivity ε_0 of free space is expressed as

(a) $Pa^{-1}V^{2}L^{-2}$ (b) $Pa^{1}V^{-2}L^{2}$ (c) $Pa^{1}V^{2}L^{-2}$ (d) $Pa^{-1}V^{-2}L^{2}$

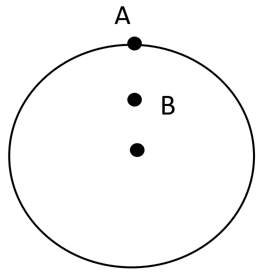
18. A cycle wheel of mass M and radius R fitted with a siren at a point on its circumference, is mounted with its plane vertical on a horizontal axle at about 3 feet above the ground. An observer stands in the vertical plane of the wheel at 100 m away from the axle of the wheel on a horizontal platform. The siren emits a sound of frequency 1000 Hz and the wheel rotates clockwise with a uniform angular speed $\omega = \pi\ rad/\sec$. Initially at t = 0 sec the siren is nearest to the observer and moves downwards. The observer records the highest pitch of sound for the first time after (speed of sound in air is 330 ms $^{-1}$)

(a) 0.30 s (b) 1.8 s (c) 2.3 s (d) 9.8 s

19. On a right angled transparent triangular prism ABC, when a ray of light is incident on face AB, parallel to the hypotenuse B**C**, it emerges out of the prism grazing along the surface AC. If instead the ray is made incident on face AC, parallel to the hypotenuse CB it gets totally reflected on face AB. The refractive index μ of the material of the prism is

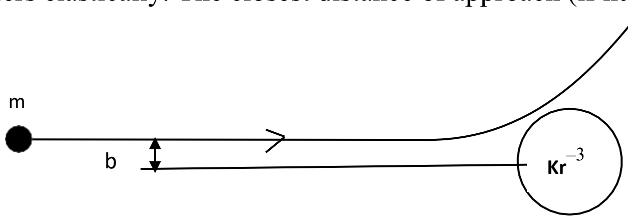
(a) $\mu > \sqrt{2}$ (b) $\sqrt{2} > \mu > \sqrt{\frac{3}{2}}$ (c) $\sqrt{3} > \mu > \sqrt{2}$ (d) $\mu < \sqrt{\frac{3}{2}}$

20. A circular disc of radius R = 10 cm is uniformly rolling on a horizontal surface with a velocity v = 4 ms^{-1} of centre of mass without slipping, the time taken by the disc to have the speed of point A (which lies on the circumference) equal to the present speed of point B (point B lies midway between centre and the point A) is



(a) t = 0.025 s (b) t = 0.036 s
(c) t = 0.046 s (d) t = 0.064 s

21. As shown in the figure, a particle of mass $m = 10^{-10} kg$, moving with velocity $v_0 = 10^5$ m/s approaches a stationary fixed target with impact parameter b from a large distance. If the fixed rigid target has a core with repulsive central force $F(r) = \frac{K}{r^3}$ where constant K > 0 and the particle scatters elastically. The closest distance of approach (if numerically K = b^2) is



(a) b (b) $b\sqrt{2}$ (c) $b\sqrt{3}$ (d) 2b

22. If the specific activity of C^{14} nuclide in a certain ancient wooden toy is known to be $\frac{3}{5}$ of that in a recently fallen tree of the same class, the age of the ancient wooden toy is
(The half life of C^{14} is 5570 years)

(a) 5570 years (b) 4105 years (c) 3342 years (d) 2785 years

In questions 23 and 24 mark your answer as

(a) If statement I is true and statement II is true and also if the statement II is a correct explanation of statement I

(b) If statement I is true and statement II is true but the statement II is a not a correct explanation of statement I

(c) If statement I is true but the statement II is false

(d) If statement I is false but statement II is true

23. Statement I: Work done in bringing a charge q from infinity to the center of a uniformly charged non – conducting solid sphere of radius R (with a total charge Q) is zero.

Statement II: The potential difference between the Centre and the surface of the uniformly charged non – conducting solid sphere of radius R (with a total charge Q) is $\frac{1}{4\pi\varepsilon_0} \times \frac{Q}{2R}$.

24. Statement I: The current flowing through a p-n junction is more in forward bias than that in the reverse bias.

Statement II: The diffusion current, dominant in forward bias, is more than the drift current, dominant in the reverse bias.

A - 2

ANY NUMBER OF OPTIONS 4, 3, 2 or 1 MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL THE CORRECT OPTIONS ARE BUBBLED

25. A simple pendulum consisting of a small bob of mass m attached to a massless inextensible string of length ℓ , hanging vertically from the ceiling, is oscillating in a vertical plane with an angular amplitude θ_m such that the maximum tension in its string is three times the minimum tension in the string i.e., $T_{max} = 3T_{min}$. The correct option(s) is/are

(a) The maximum tension in the string is $T_{max} = mg\,(3 - 2\cos\theta_m)$

(b) The maximum tension in the string is $T_{max} = \frac{9}{5} mg$

(c) The maximum velocity of the bob on its way is $v_{max} = 3.96\ ms^{-1}$

(d) The angular amplitude θ_m lies in the range $\frac{\pi}{4} < \theta_m < \frac{\pi}{3}$

26. Two small masses m and M lie on a large horizontal frictionless circular track of radius R. The two masses are free to slide on the track but constrained to move along a circle. Initially the two masses are tied by a thread with a compressed spring between them (spring of negligible length being attached with none of the two masses). The compressed spring stores a potential energy U_0. At a certain time t = 0 the thread is burnt and the two masses are released to run opposite to each other leaving the spring behind. The total mechanical energy remaining conserved. On the circular track the two masses make a head on perfectly elastic collision. Take M = 2m for all calculations. Which of the following option(s) is / are correct?

(a) The angle turned by mass m before the collision is $\theta = 4\frac{\pi}{3}$

(b) The velocity of mass m on the track is $u = \sqrt{\frac{4U_0}{3m}}$

(c) The time taken to collide for the first time is $t_1 = 2\pi R\sqrt{\frac{m}{3U_0}}$

(d) The time taken for the second collision is $t_2 = 2\pi R\sqrt{\frac{2m}{3U_0}}$

27. The electric field component of an electromagnetic wave is expressed as $E = (3j + bk)\times 10^{-3}\sin\left[10^7\left(x + 2y + 3z - \beta t\right)\right]$ in SI units. Taking $c = 3\times 10^8\ ms^{-1}$ as the speed of electromagnetic wave in vacuum, choose the correct option(s)

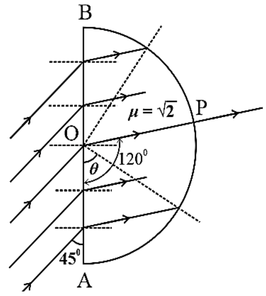
(a) The value of constant beta is $\beta = 3\times 10^8 \times \sqrt{14}$

(b) The value of constant b is b = 2.

(c) The average energy density of the em wave is $U = 6.5\times 10^{-6}\,\varepsilon_0$ in SI units.

(d) The amplitude of magnetic field is $B = 1.20\times 10^{-11}\ Tesla$

28. A parallel beam of light is made incident (as shown) on the flat diametric plane of a transparent semi-circular thin sheet of thickness t (t << R) of refractive index $\mu = \sqrt{2}$ at an angle of 45^0. As a result of refraction, the light enters the semi-circular sheet and comes out at its curved surface.



(a) Light rays come out at the curved surface for values of θ in the range $75^0 \le \theta \le 165^0$.
(b) The range of angle θ is independent of the angle of incidence.
(c) The range of angle θ depends on the refractive index of the material
(d) All the emergent rays of light shall cross the line OP which is a refracted ray at $\theta = 120^0$

Here θ is the angle between the vertical diameter AB and the concerned radius of the semicircular sheet of radius R.

29. A certain rod of uniform area of cross section A (A = 1.0 cm^2) with its length = 2 m is thermally insulated on its lateral surface. The thermal conductivity (K) of the material of the rod varies with temperature T as $K = \frac{\alpha}{T}$ where α is a constant. The two ends of the rod are maintained at temperature of $T_1 = 90^\circ$ ‿ and $T_2 = 10^\circ$ ‿. The correct option(s) is /are

(a) The temperature at 50 cm from the colder end is 17.32° ‿
(b) The temperature at 50 cm from the hotter end is 51.96° ‿
(c) The rate of heat flow per unit area of cross section of the rod is 1.1α in SI units.
(d) The temperature gradient is numerically higher near the hot end compare to that near the cold end.

30. Positronium is a short-lived $(\approx 10^{-9}\text{s})$ bound state of an electron and a positron (a positively charged particle with mass and charge equal (in magnitude) to an electron) revolving round their common centre of mass. If E_0, v_0 and a_0 are respectively the ground state energy, the orbital speed of electron in first orbit and the radius of the first (n = 1) Bohr orbit for Hydrogen atom, the corresponding quantities E, v and a for the positronium are

(a) $E = \frac{E_0}{2}$ (b) $a = a_0$ (c) $a = 2a_0$ (d) $E = E_0, v = v_0, a = a_0$

31. A thin double convex lens of radii of curvature R_1 = 20 cm and R_2 = 60 cm is made-up of a transparent material of refractive index μ = 1.5. Choose the correct option(s)

(a) The focal length of the lens is f = 30 cm when in air.
(b) The lens behaves as a concave mirror of focal length $f_M = 10\ cm$ when silvered on the surface of radius R_2 = 60 cm
(c) The lens behave as a concave lens (diverging lens) if the image space beyond R_2 = 60 cm radius surface is filled with a transparent liquid of refractive index $\mu = \frac{5}{3}$. The object space prior to the surface of radius R_1 = 20 cm is air.
(d) A beam of rays incident parallel to principal axis focuses at 48 cm behind the lens if water $\left(\mu = \frac{4}{3}\right)$ fills the entire space behind the surface of radius R_2 = 60 cm. The object space prior to the surface of radius R_1 = 20 cm is air.

32. A thick hollow cylinder of height h and inner and outer radii a and b (b > a) made up of a poorly conducting material of resistivity ρ lies coaxially inside a long solenoid at its middle. The radius of the solenoid is larger than b. Throughout the interior of the solenoid, a uniform time varying magnetic field $B = \beta t$ is produced parallel to solenoid axis. Here β is a constant. In this time varying magnetic field

(a) the emf induced at a certain radius r (a < r < b) in the hollow cylinder is $\pi r^2 \beta$
(b) the induced current circulating in the thick hollow cylinder between radii a and b is $i = \frac{\beta h}{4\rho}\left(b^2 - a^2\right)$
(c) the resistance offered to the circulation of current by the thick hollow cylinder is $R = \frac{2\pi\rho}{h \times \ln\frac{b}{a}}$
(d) no electric field is detectable outside the solenoid.

ROUGH WORK

INDIAN OYMPIAD QUALIFIER IN PHYSICS 2021-22 (PART- I)

IOQP 2021-22 PART I (NSEP) Held on March 13, 2022

FINAL ANSWER KEY FOR IOQP 2021-22 PART 1

Question	PAPER CODE 61	PAPER CODE 62	PAPER CODE 63	PAPER CODE 64
1	c	d	b	b
2	b	b	b	a
3	d	c	b	b
4	d	b	b	b
5	d	b	b	a
6	a	b	d	d
7	a	b	a	b
8	a	b	c	c
9	b	b	b	b
10	a	d	d	b
11	b	a	d	b
12	b	c	d	b
13	a	b	a	b
14	d	d	a	b
15	b	d	a	d
16	c	d	b	a
17	b	a	a	c
18	b	a	b	b
19	b	a	b	d
20	b	b	a	d
21	b	a	d	d
22	b	b	b	a
23	d	b	c	a
24	a	a	b	a
25	a, b, c, d	a, c, d	a, b, d	a, c, d
26	a, b, c	a, b, c, d	a, b, c	a, c, d
27	a, c, d	a, c	a, b, c, d	a, b, c, d
28	a, c, d	a, b, d	a, b, c	a, c
29	a, b, c, d	a, b, c	a, c, d	a, b, d
30	a, c	a, b, c, d	a, c, d	a, b, c
31	a, b, d	a, b, c	a, b, c, d	a, b, c, d
32	a, b, c	a, c, d	a, c	a, b, c

IOQP 2021-22 PART I (NSEP – 2021-22) Solution– 61

1. The instantaneous rate of absorption of heat is $\frac{dQ}{dt} \propto 4\pi r^2 = k4\pi r^2$ Also $\frac{dQ}{dt} = -L\frac{dm}{dt}$ So $-L\frac{dm}{dt} = k4\pi r^2$ *or* $-L\frac{d}{dt}\left(\frac{4\pi}{3}r^3\rho\right) = k4\pi r^2$ or $\frac{dr}{dt} = -\frac{k}{\rho L} \Rightarrow \int_{r_0}^{r} dr = -\frac{k}{\rho L}\int_0^t dt \Rightarrow r = r_0 - \frac{k}{\rho L}t$ or $r = -\frac{k}{\rho L}t + r_0$ which is a straight line with negative slope $= -\frac{k}{\rho L}$ where k is constant. **Ans: c**

2. The process from A to B is isochoric $\because P \propto T$ means the volume is constant. Therefore the work done $dW_{AB} = PdV = 0$ From B to C the process is isobaric so work done is $dW_{BC} = PdV = nRdT = 3\times R\times(600-200) = 1200R$ .CD is again isochoric process so work done $W_{CD} = 0$. Further the process from D to A is isobaric means P constant and work done is $dW_{DA} = PdV = nRdT$ or $dW_{DA} = 3\times R(100-300) = -600R$. Thus the total work done is $W = 1200R - 600R = 600R = 4986J = 4.986kJ = 5.0kJ$ **Ans: b**

3. From first law of thermodynamics $dQ = dU + dW \Rightarrow C\,dT = C_V\,dT + P\,dV$. Given that $C = C_V + \alpha\frac{P}{T}$ Substituting the value we get $\left(C_V + \alpha\frac{P}{T}\right)dT = C_V dT + PdV \Rightarrow \alpha\frac{dT}{T} = dV$ on integration we get $\alpha \times \ell nT = V + \text{constant}$ or $T = Ae^{V/\alpha}$ **Ans: d**

4. The magnetic field produced by a current carrying conductor at a distance x is $B = \frac{\mu_0 I}{2\pi x}$ Therefore the induced emf in a conductor of length dx moving with velocity v is $d\varepsilon = -\ell Bv = -\frac{\mu_0 I}{2\pi x}v\,dx$. Total emf produced in the present problem is
$$\varepsilon = -\int_{2\ell}^{3\ell}\frac{\mu_0 I}{2\pi x}v\,dx = -\frac{\mu_0 I}{2\pi}v\int_{2\ell}^{3\ell}\frac{dx}{x} = \frac{\mu_0 I}{2\pi}v \times \ln 1.5$$ **Ans: d**

5. Since both the charges are positive, the electric field at any point between them is $E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x^2} - \frac{Q}{(L-x)^2}\right]$ This will be positive for $0 < x < \frac{L}{2}$ and negative for $\frac{L}{2} < x < L$ as shown in figure d. The curve is a $\frac{1}{x^2}$ type. **Ans d**

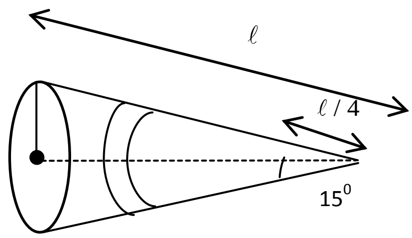
6. When the current flows in the wire along AB, the magnetic field in the circular loop is directed outward perpendicular to the plane of the paper. During the increase of current i.e., from 0 to T/2, the induced current in the loop is clockwise while during the decrease of current i.e., T/2 to T the induced current shall be anticlockwise. Hence the answer is a.
Ans a

7. The $2.0 M\Omega$ resistance is connected in series with R_B and the cell. When we connect the voltmeter of resistance $r\,M\Omega$ in parallel to $2.0 M\Omega$ we get $3 = \frac{9}{\frac{2r}{r+2} + R_B} \times \frac{2r}{r+2}$

$\Rightarrow 2r + R_B(r+2) = 6r \Rightarrow R_B = \frac{4r}{r+2}$ When we connect the same voltmeter in parallel with R_B we get $4.5 = \frac{9}{2 + \frac{rR_B}{r+R_B}} \times \frac{rR_B}{r+R_B} \Rightarrow 2(r + R_B) + rR_B = 2rR_B \Rightarrow R_B = \frac{2r}{r-2}$ Comparing the result $\frac{4r}{r+2} = \frac{2r}{r-2} \Rightarrow r = 6$ Thus $r = 6 M\Omega$ and $R_B = 3\,M\Omega$ **Ans: a**

8. The magnifying power of a compound microscope when the final image is formed at D, the least distance of distinct vision is $MP = \frac{V_0}{U_0}\left(1 + \frac{D}{f_e}\right)$ Now as per the given conditions $50 = \frac{V_0}{U_0}\left(1 + \frac{25}{5}\right) \Rightarrow \frac{V_0}{U_0} = \frac{25}{3}$ Now for the objective lens $\frac{1}{V_0} - \frac{1}{-U_0} = \frac{1}{f_0} \Rightarrow 1 + \frac{V_0}{U_0} = \frac{V_0}{f_0}$ comparing the two, we get $V_0 = \frac{28}{3} cm.$ Increasing the length of microscope by 2 cm, Then $V_0 = \frac{28}{3} + 2 = \frac{34}{3} cm$ In the new situation $1 + \frac{V_0'}{U_0'} = \frac{V_0'}{f_0} = \frac{34}{3} \Rightarrow \frac{V_0'}{U_0'} = \frac{31}{3}$ The magnifying power therefore now becomes $MP = \frac{31}{3}\left(1 + \frac{25}{5}\right) = 62$ **Ans: a**

9. Let us consider an elementary ring of width $d\xi$ at a slant distance ξ from the vertex of the cone. The charge on the circular ring shall be $dq = 2\pi\xi \sin\theta\, d\xi \times \sigma$. The electric field produced by this elementary ring at the vertex of the cone is $dE = \frac{1}{4\pi\varepsilon_0} \times \frac{2\pi\xi \sin\theta\, d\xi \times \sigma \times \xi\cos\theta}{\xi^3}$ Thereby the electric field



E at the vertex shall be $E = \frac{\sigma}{4\varepsilon_0} 2\sin\theta\cos\theta \int_{l/4}^{l} \frac{d\xi}{\xi} \Rightarrow E = \frac{\sigma}{4\varepsilon_0}\sin 2\theta \{\ln\xi\}_{l/4}^{l}$

$\Rightarrow E = \frac{\sigma}{8\varepsilon_0} \times 2\ln 2 = \frac{\sigma}{4\varepsilon_0}\ln 2$ **Ans: b**

10. According to the Newton's Second Law $F = \frac{dp}{dt}$. In the present case the rain drop is attracted by the earth so at any instant, $mg = \frac{d}{dt}(mv) \Rightarrow mg = m\frac{dv}{dt} + v\frac{dm}{dt}$ *or* $g = \frac{dv}{dt} + \frac{v}{m}\frac{dm}{dt}$

Given that $\frac{dm}{dt} = kt^2 \Rightarrow m = \frac{kt^3}{3} + m_0$ where m_0 is initial mass. Further

$$g = \frac{dv}{dt} + \frac{v}{\frac{kt^3}{3} + m_0} \times kt^2 \Rightarrow \frac{dv}{dt} = g - \frac{3kt^2}{3m_0 + kt^3} \times v \Rightarrow \frac{dv}{dt} + \left(\frac{3kt^2}{3m_0 + kt^3}\right)v = g \text{ or}$$

$$\Rightarrow \left(3m_0 + kt^3\right)dv + v3kt^2 dt = g\left(3m_0 + kt^3\right)dt \text{ or} \Rightarrow d\left\{\left(3m_0 + kt^3\right)v\right\} = g\left(3m_0 + kt^3\right)dt$$

Integrating we get $\left(3m_0 + kt^3\right)v = \left(3m_0 gt + \frac{gkt^4}{4}\right)_0^t$ or

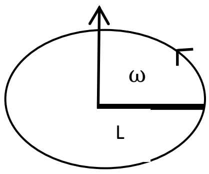
$$v = g\frac{12m_0 t + kt^4}{4\left(3m_0 + kt^3\right)} = \frac{1905}{1506}g = 12.4\,ms^{-1}$$ **Ans: a**

11. Two planets of mass M and radius R each are separated by distance 4R. A mass m has to be thrown from Planet A so as just to reach Planet B. For this we need to throw the mass so that it just reaches the midpoint then after it will be attracted by B. The potential energy of the mass m on the surface of the planet A is $U_A = -\frac{GMm}{R} - \frac{GMm}{3R} = -\frac{4GMm}{3R}$ and the potential energy at the midpoint between the two planets is $U_{Mid} = -\frac{GMm}{2R} - \frac{GMm}{2R} = -\frac{GMm}{R}$

Hence the energy needed to project the body is

$$\frac{1}{2}mv^2 = \left(-\frac{GMm}{R}\right) - \left(-\frac{4GMm}{3R}\right) \Rightarrow v = \sqrt{\frac{2GM}{3R}}$$ **Ans: b**

12. When the rod rotates about a vertical axis through one of its ends, every pointon the rod experiences a centrifugal force. If we consider a small length dx of mass $\lambda\,dx$ at a distance x from the axis where $\lambda = \frac{M}{L} = \frac{\pi R^2 L\rho}{L} = \pi R^2\rho$,



The outward pull on this length x is $= \frac{\lambda\,dx\,\omega^2\,x^2}{x} = T\left(say\right)$

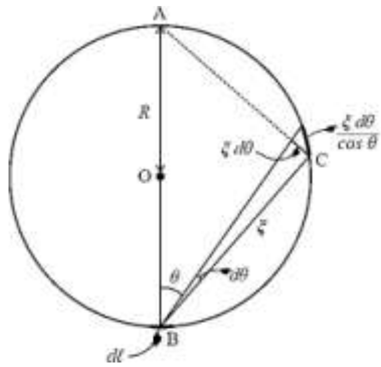
This force will cause an elongation in the rod, because of its elasticity.

The elongation may be given by $d\xi = \frac{T\,x}{AY} = \frac{\lambda\,dx\,\omega^2\,x}{\pi R^2 Y}.x = \frac{\rho\omega^2}{Y}.x^2\,dx.$

The total elongation in the rod is therefore $\int d\xi = \frac{\rho\omega^2}{Y}\int_0^L x^2\,dx = \frac{\rho\omega^2 L^3}{3Y}.$

Ans: b

13. Since the total energy is fixed, the kinetic energy so to say the magnitude of the momentum will be large where ever the potential energy is less and vice versa. Further the momentum $p = \pm\sqrt{2m\times\left(KE\right)} = \pm\sqrt{2m\times\left(E-V\right)}$ .Here (E-V) is the kinetic energy of the particle. The curve for momentum will be symmetric about x axis so curve a.
Ans: a



14. In a hollow metallic cylinder current is sent parallel to its axis along the entire curved surface. Let us consider a thin strip of width dl on the surface and along the length of the cylinder at a point B and another parallel strip of width $\frac{\xi d\theta}{\cos\theta}$ at the end of the chord of length ξ at angle θ . If I be the current through the metallic cylinder then the current per unit width shall be $j = \frac{I}{2\pi R}$. Thereby the current through the two parallel strips separated by ξ shall be $jdl\ and\ j\frac{\xi d\theta}{\cos\theta}$ respectively.

The force of attraction between these two parallel wires shall thus be $F = \frac{\mu_0}{2\pi}\frac{jdl\times j\xi d\theta}{\xi\cos\theta}\ Nm^{-1}$.Component of this force towards the centre will add togive resultant inward force then dividing by dl and integrating over θ within the limits $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$ we obtain force per unit surface of the hollow cylinder. Thusthe inward force per unit surface area is

$$=\sum F\cos\theta = \frac{\mu_0 j^2}{2\pi}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta = \frac{\mu_0 j^2}{2} = \frac{\mu_0}{2}\left(\frac{I}{2\pi R}\right)^2 = 2.55\times10^{-7}\,Nm^{-2}$$

Ans: d

15. By the principle of conservation of momentum along x-direction $2Mv\cos\theta + M\frac{3}{2}v\cos\phi = 2M\times\frac{4}{5}v + MV_X$ where V_x is the velocity of mass M in x direction after the collision. Substituting the values $2Mv\times\frac{4}{5} + M\frac{3}{2}v\times\frac{3}{5} = 2M\times\frac{4}{5}v + MV_X \Rightarrow V_X = \frac{9}{10}v$

In y-direction the momentum conservation yields $-2Mv\sin\theta + M\frac{3}{2}v\sin\phi = 2M\times0 + MV_y$

substituting the values $-2Mv\frac{3}{5} + M\times\frac{3}{2}v\times\frac{4}{5} = 0 + MV_y \Rightarrow V_y = 0$ means no velocity in y-direction. Hence the centre of mass moves along x-direction. The velocity of centre of mass after the collision is $V_{xCM} = \frac{2M\times\frac{4}{5}v + M\times\frac{9}{10}v}{3M} = \frac{25}{30}v = \frac{5}{6}v$ and not $\frac{5}{2}v$ Also the linear momentum before collision is $\frac{25}{10}Mv = \frac{5}{2}Mv$ and not $\frac{5}{6}Mv$ **Ans: b**

16. Let us consider that the height of the liquid surface in the hemispherical bowl is h at a certain time t. The radius of water surface at this time shall be $=\sqrt{R^2-(R-h)^2}$ So the surface area of the liquid at this time will be $=\pi\left\{R^2-(R-h)^2\right\}=\pi\left(2Rh-h^2\right)$ Further considering that the liquid height falls through dh in time dt, the volume of liquid flowing out per second can be written as $-\pi\left(2Rh-h^2\right)\frac{dh}{dt}=va=a\sqrt{2gh}$ Thereby $dt=-\frac{\pi}{a\sqrt{2g}}\left(2Rh^{1/2}-h^{3/2}\right)dh$ integrating we get $\int_0^t dt=-\frac{2\pi R}{a\sqrt{2g}}\int_{R/2}^{0}h^{1/2}dh+\frac{\pi}{a\sqrt{2g}}\int_{R/2}^{0}h^{3/2}dh \Rightarrow t=\frac{17\pi R^2}{60a}\sqrt{\frac{R}{g}}$

Ans: c

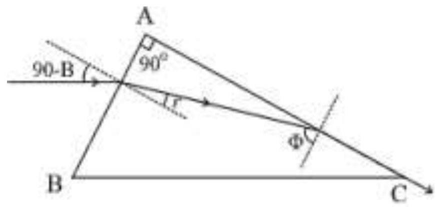
17. To obtain dimensional formula for ε_0 let us express Coulomb's law as

$$F=\frac{1}{4\pi\varepsilon_0}\times\frac{q_1q_2}{r^2}=4\pi\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_1}{r}\right)\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_2}{r}\right)\Rightarrow\left(\frac{F}{4\pi r^2}\right)r^2\cong\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_1}{r}\right)\left(\frac{1}{4\pi\varepsilon_0}\times\frac{q_2}{r}\right)$$

$\Rightarrow Pa\times L^2=\varepsilon_0V^2\Rightarrow\varepsilon_0=\left(Pa\right)^1L^2V^{-2}$ **Ans: b**

18. The observer will record the maximum frequency $\upsilon'=\frac{v}{v-v_s}\times\upsilon$ when the sound produced by the siren, in the top most point of the circumference of the wheel, reaches the listener. This sound will reach the listener in time $t=\frac{100}{330}=0.303\text{s}$ after being produced. Also the wheel itself will take time $t_0=\frac{3}{4}\times\frac{2\pi}{\omega}$ substituting $\omega=\pi$ we get $t_0=\frac{3\pi}{2\pi}=1.5\ s$ Hence the total time is $t+t_0=0.303+1.5=1.803s$ **Ans: b**

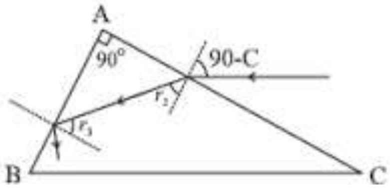
19. For a ray of light incident on side AB parallel to the base, we can write that the refractive index



$$\mu = \frac{\sin(90-B)}{\sin r} = \frac{\cos B}{\sin r} \text{ or } \mu \sin r = \cos B....(1)$$

Also $r+\phi = 90$ or $\sin r = \sin(90-\phi) = \cos\phi = \sqrt{1-\sin^2\phi}$

or $\sin r = \sqrt{1-\frac{1}{\mu^2}} = \sqrt{\frac{\mu^2-1}{\mu^2}}$ or $\mu \sin r = \sqrt{\mu^2-1}$ ...(2)



From (1) and (2) $\cos B = \sqrt{\mu^2-1}$

or $\cos^2 B = (\mu^2-1)$....(3)

Next the ray is incident parallel to base on the side AC.

Here also $r_2 + r_3 = 90$ and since $r_3 > \phi$ i.e.the critical angle Now using $r_2 = 90 - r_3$ and

$\mu = \frac{\sin(90-C)}{\sin r_2} = \frac{\sin(90-C)}{\sin(90-r_3)} = \frac{\cos C}{\cos r_3}$ or $\mu\cos r_3 = \cos C = \cos(90-\mathrm{B}) = \sin \mathrm{B}$ This in turn gives $\mu^2(1-\sin^2 r_3) = \sin^2 B$ .... (4) adding equation (3) and (4)

$$\left(\mu^2 - \mu^2\sin^2 r_3\right) + \mu^2 - 1 = 1 \Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right) = \sin^2 r_3....(5)$$

Further angle $r_3 > \phi$ or $\sin r_3 > \sin\phi$

$\Rightarrow \sqrt{2\left(\frac{\mu^2-1}{\mu^2}\right)} > \sin\phi \Rightarrow 2\left(\frac{\mu^2-1}{\mu^2}\right) > \frac{1}{\mu^2} \Rightarrow \mu^2 - 1 > \frac{1}{2} \Rightarrow \mu^2 > \frac{3}{2} \Rightarrow \mu > \sqrt{\frac{3}{2}}$ Also

$r+\phi = 90$ *but* $r < \phi$ So essentially $\phi > 45$ or $\sin\phi > \sin 45$ *or* $\frac{1}{\mu} > \frac{1}{\sqrt{2}} \Rightarrow \mu < \sqrt{2}$ Thus

we can conclude that $\sqrt{\frac{3}{2}} < \mu < \sqrt{2}$ **Ans: b**

20. Under the conditions of pure rolling of the disc, the velocity of the point A (at the top) on the circumference is $v + \omega R = 2v$ where as the velocity of point B at half the radius is $v + \omega\frac{R}{2} = \frac{3}{2}v$ Let the final speed of point A becomes $\frac{3}{2}v$ after turning through an angle ϕ

then $\frac{3}{2}v = \sqrt{v^2 + \omega^2R^2 + 2v\omega R\cos\phi} = \sqrt{v^2+v^2+2v^2\cos\phi} \Rightarrow \frac{3}{2} = \sqrt{2+2\cos\phi}$ or

$\Rightarrow \frac{3}{2} = \sqrt{2+2\left(2\cos^2\frac{\phi}{2}-1\right)} = 2\cos\frac{\phi}{2}$ *or* $\cos\frac{\phi}{2} = \frac{3}{4} \Rightarrow \phi = 82.82^\circ$ Further if $T = \frac{2\pi R}{v}$ be the time period then by simple unitary method time taken to turn through $\phi = 82.82^\circ$ is

$t = \frac{\phi}{360} \times \frac{2\pi R}{v} = 0.036\ s$ **Ans: b**

21. At the point of closest approach (distance) the particle will have tangential velocity expressed as $v_t = \omega d = \dot{\theta} d$ By conservation of angular momentum $mvb = I\omega = md^2 \dot{\theta} \Rightarrow \dot{\theta} = \omega = \frac{vb}{d^2}$

This being a case of elastic scattering, the conservation of energy provides

$\frac{1}{2}mv^2 + 0 = \frac{1}{2}md^2\omega^2 - \int \frac{K}{r^3}.\, dr \Rightarrow mv^2 = md^2\omega^2 + \frac{K}{d^2} \because PE = -\int \frac{K}{r^3} dr = \frac{K}{2r^2} = \frac{K}{2d^2}$

Thereby $mv^2 = md^2\left(\frac{vb}{d^2}\right)^2 + \frac{K}{d^2} = \left(mv^2b^2 + K\right)\frac{1}{d^2}$ Substituting $m = 10^{-10}\, Kg$,

$v = 10^5\, ms^{-1}$ and numerically $K = b^2$ we obtain $d^2 = \left(mv^2b^2 + k\right)\frac{1}{mv^2} = 2b^2 \Rightarrow d = b\sqrt{2}$

Ans: b

22. According to the law of radioactive disintegration $N = N_0 e^{-\lambda t}$ The activity therefore is $-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$ Given that at certain time t the activity of the sample is $\left(-\frac{dN}{dt}\right)_t = \frac{3}{5}\left(-\frac{dN}{dt}\right)_{t=0} = \frac{3}{5}\lambda N_0$ So $e^{-\lambda t} = \frac{3}{5} \Rightarrow \lambda t = \ln\left(\frac{5}{3}\right)$ or $\frac{\ln 2}{T} \times t = \ln\left(\frac{5}{3}\right)$

$\Rightarrow t = \frac{T \times \ln(5/3)}{\ln 2} = \frac{5570 \times 0.5108}{0.693} = 4105\, years$ **Ans: b**

23. The statement I is false but the statement II is true hence **Ans: d**

24. The statement I is true and the statement II is also true. Also the statement II is the cause of I hence **Ans: a**

25. In a swinging simple pendulum, the tension in the string at any arbitrary position may be expressed as $T = mg\cos\theta + \frac{mv^2}{l}$ The conservation of energy provides

$\frac{1}{2}mv^2 = mg\left(l\cos\theta - l\cos\theta_m\right)$ thereby $\frac{mv^2}{l} = 2mg\left(\cos\theta - \cos\theta_m\right)$ therefore the tension becomes $T = mg\left(3\cos\theta - 2\cos\theta_m\right)$ Obviously the tension depends on the angle θ and will be maximum when $\theta = 0$ So the maximum tension is $T_{\max} = mg\left(3 - 2\cos\theta_m\right)$ and the minimum tension (when $\theta = \theta_m$) is $T_{\min} = mg\cos\theta_m$ According to the given condition

$T_{\max} = 3T_{\min}$ Hence $mg\left(3 - 2\cos\theta_m\right) = 3mg\cos\theta_m \Rightarrow \cos\theta_m = \frac{3}{5} \Rightarrow \theta_m = 53.13^0$

$\Rightarrow \frac{\pi}{4} \le \theta_m \le \frac{\pi}{3}$ and the maximum tension is $T_{\max} = \frac{9}{5}mg.$ The maximum velocity

$v_{\max}^2 = 2gl\left(1 - \cos\theta_m\right) = 2gl\left(1 - \frac{3}{5}\right) = \frac{4gl}{5} \Rightarrow v_{\max} = \sqrt{\frac{4gl}{5}} = 3.96\, ms^{-1}$ **Ans: a,b,c,d**

26. When the masses are released, they move in opposite direction with equal momentum i.e., $mv + MV = 0 \Rightarrow V = -\frac{mv}{M}$ ...(1) Numerically $V = \frac{mv}{M}$ Let the two masses collide for the first time after time t and the mass m turns through angle θ during this period then $t = \frac{\theta}{\omega_1} = \frac{2\pi - \theta}{\omega_2}$ or $t = \frac{\theta R}{v} = \frac{(2\pi - \theta)R}{V} \Rightarrow \left(\frac{1}{v} + \frac{1}{V}\right)\theta = \frac{2\pi}{V}$ Substituting the value of V, we obtain $\theta = \frac{2\pi M}{m+M} = \frac{4\pi}{3}$ if M = 2m. Also Energy conservation provide that

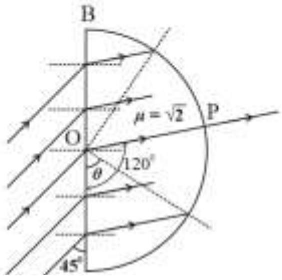
$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = U_0 \quad or \quad \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2 = U_0 \text{ or } \frac{1}{2}mv^2\left[1 + \frac{m}{M}\right] = U_0$$

$\Rightarrow v = \sqrt{\frac{2MU_0}{m(\mathrm{m}+\mathrm{M})}} = \sqrt{\frac{4U_0}{3m}}$ if M = 2m Thus the time taken for firstcollision is

$t = \frac{\theta R}{v} = \frac{4\pi R}{3\sqrt{\frac{4U_0}{3m}}} = 2\pi R\sqrt{\frac{m}{3U_0}}$ Lastly the time taken for the second collision must be just double of it and not $2\pi R\sqrt{\frac{2m}{3U_0}}$ **Ans: a, b, c**

27. Given that the Electric field $\vec{E} = (3\hat{j} + b\hat{k}) \times 10^{-3} \sin[10^7(\mathrm{x} + 2\,\mathrm{y} + 3\mathrm{z} - \beta\,\mathrm{t})]$ Knowing $\vec{K}.\vec{r} = (\hat{\imath}k_x + \hat{j}k_y + \hat{k}k_z).(\hat{\imath}x + \hat{j}y + \hat{k}z) = xk_x + yk_y + zk_z$. Comparing it with the given expression we get $xk_x + yk_y + zk_z = 10^{+7}(\mathrm{x} + 2\,\mathrm{y} + 3\,\mathrm{z})$ Thereby $\Rightarrow k_x = 10^7,\ k_y = 2 \times 10^7 \ \& \ k_z = 3 \times 10^7$ or the vector $\vec{K} = (\hat{\imath} + 2\hat{j} + 3\hat{k}) \times 10^7$ $\Rightarrow K = 10^7\sqrt{14}$ Also the speed of the wave $c = \frac{\beta}{K}$ $\therefore\ \beta = c \times 10^7\sqrt{14} = 3 \times 10^{15}\sqrt{14}$ Furtherfor any electromagnetic wave $\vec{k} \cdot \vec{E} = 0$ Therefore $10^{+7}(\hat{\imath} + 2\hat{j} + 3\hat{k}).(3\hat{j} + b\hat{k}) \times 10^{-3} = 0 \Rightarrow 2 \times 3 + 3b = 0 \Rightarrow b = -2$ this makes option b wrong. Further the energy of an em wave is $= \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\sqrt{3^2 + 2^2}\right)^2 \times 10^{-6} = 6.5\varepsilon_0\ \mu J$. The magnetic field can be obtained as $B = \frac{E}{c} = \frac{10^{-3}\sqrt{13}}{3 \times 10^8} = 1.20 \times 10^{-11}\,Tesla$ **Ans: a,c,d**

28. Snell's law is $\mu = \dfrac{\sin i}{\sin r} \Rightarrow \sqrt{2} = \dfrac{\sin 45}{\sin r} \Rightarrow \sin r = \dfrac{1}{2} \Rightarrow r = 30^0$



The critical angle is $\sin^{-1}\dfrac{1}{\sqrt{2}} = 45^0$.

For the emergence, theangle of incidence at curved surface must be less than 45^0 therefore angle θ should be greater than

$$\theta_{\min} = 180 - \left(90 - \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) - \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{\min} = [180 - (90 - 30) - 45] = 75^0$$

Towards the upper edge angle θ must be less than

$$\theta_{\max} = \left(90 + \sin^{-1}\left(\frac{\sin i}{\mu}\right)\right) + \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\theta_{\max} = (90 + 30) + 45 = 165$$

Thus we obtain $\theta_{\min} < \theta < \theta_{\max}$ *as* $75^0 \le \theta \le 165^0$ for the emergence of light through the curved surface.Thus the light will come out only for the angle θ lying within the range $\theta_{\max} - \theta_{\min} = 2\sin^{-1}\left(\dfrac{1}{\mu}\right)$ which is independent of the angle of incidence but depends on the refractive index (μ) of the material. Of course the range is same for all values of angle of incidence yet the values of $\theta_{\min}$ *and* $\theta_{\max}$ are different for different values of angle of incidence. Hence option **b is not correct**. The light coming out of the curved surface will go away from normal hence towards the line which is the increased radius for $\theta = 120^0$ and thus form a convergent beam towards the enhanced radius corresponding to $\theta = 120^0$.

Ans: a, c, d

29. The rate of flow of heat in a solid rod is expressed as the thermal current

$H = \dfrac{dQ}{dt} = KA\left(-\dfrac{dT}{dx}\right)$ Given that the thermal conductivity $K = \dfrac{\alpha}{T}$ so one can write

$H\int_0^l dx = -\alpha A\int_{90}^{10} \dfrac{dT}{T} \Rightarrow Hl = -\alpha A \ln\dfrac{10}{90} = 2\alpha A \ln 3$ using $l = 2m$ we get

$H = \alpha A \ln 3 = 1.1\alpha A$ Further at any intermediate location at a distance x from hot end

$$H\int_0^x dx = -\alpha A\int_{90}^{T} \frac{dT}{T} \Rightarrow Hx = -\alpha A \ln\frac{T}{90} \Rightarrow T = 90 \times e^{-Hx/\alpha A}$$

$At\ x = 0.5\ m,\ T = 90 \times e^{-Hx/\alpha A} = 90e^{-1.1 \times 0.5} = 51.96^0\ C$ Also

$At\ x = 1.5m,\ T = 90e^{-1.1 \times 1.5} = 17.32^0 C$ The temperature gradient may be expressed as

$\dfrac{dT}{dx} = -\dfrac{TH}{\alpha A}$ is higher near the hotter end than that near the colder end.

Ans: a, b, c, d

30. According to Bohr theory, in an hydrogen atom an electron revolves round the proton, the centripetal force being provided by electrostatic attraction. Such that

$$\frac{mv^2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} \quad \text{or } mv^2r=Ke^2 ---(1)$$

where $v\, and\, r$ are the velocity of electron and the radius of the orbit and $K=\frac{1}{4\pi\varepsilon_0}$

According to Bohr quantum condition (second postulate)

$$mvr=n\frac{h}{2\pi} \;\; or \;\; mvr=n\hbar---(2)$$

Dividing eq (1) by eq (2) $v=\frac{K e^2}{n\hbar} \Rightarrow$ v does not depend on mass. Now substituting v in equation (2)

$$m\left(\frac{K e^2}{n\hbar}\right)r=n\hbar \Rightarrow r=\frac{n^2\hbar^2}{Kme^2} \text{ showing that } r\propto\frac{1}{m}$$

The total energy E = KE + PE

$$=\frac{1}{2}mv^2-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}=\frac{1}{2}m\left(\frac{K e^2}{n\hbar}\right)^2-\frac{K e^2}{r}$$

$$E=\frac{1}{2}\frac{mK^2e^4}{n^2\hbar^2}-\frac{K e^2}{n^2\hbar^2}Kme^2 \;\; E_n=-\frac{1}{2}\frac{mK^2e^4}{n^2\hbar^2}\Rightarrow E\propto m$$

To understand the situation more specifically, one replaces the mass of electron by the effective mass i.e. the reduced mass (μ) which for the case of hydrogen atom is

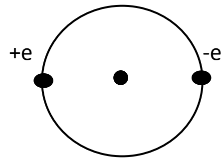
$\mu=\frac{m\times 1836m}{m+1836m}\approx m(\text{electron mass})$ Thus for hydrogen atom the

Radius of first orbit $r=a_0=\frac{\hbar^2}{Kme^2}$ we have substituted n = 1

Velocity of electron in first orbit $v=v_0=\frac{K e^2}{\hbar}$

Energy of electron in first orbit E= $E_0=-\frac{1}{2}\frac{mK^2e^4}{\hbar^2}\Rightarrow E\propto m$

A positronium is a short lived atomic entity in which a negatively charged electron is said to revolve round a positron (a positive particle having charge and mass equal to an electron even sometimes known as a positive electron) Since the particles have equal mass, the rotation takes place around the centre of mass which lies midway between the two.



In case of positronium the reduced mass is $\mu=\frac{mm}{m+m}=\frac{m}{2}$

Thereby the radius becomes $r=a=\frac{\hbar^2}{K(m/2)e^2}=\frac{2\hbar^2}{Kme^2}=2a_0$

And energy becomes $E=-\frac{1}{2}\frac{(m/2)K^2e^4}{n^2\hbar^2}\Rightarrow E=-\frac{1}{4}\frac{mK^2e^4}{n^2\hbar^2}=\frac{E_0}{2}$ **Ans: a, c**

31. The focal length f_2 of a lens of refractive index μ and radii of curvature R_1 and R_2 when the refractive index of the object space is μ_1 and that of image space is μ_2 is calculated by

$\frac{\mu_2}{f_2} = \frac{\mu - \mu_1}{R_1} - \frac{\mu - \mu_2}{R_2}$ If the lens is kept in air $\mu_1 = 1$ and $\mu_2 = 1$ then

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-60}\right) = \frac{1}{30} \Rightarrow f = 30\,cm$ Hence a is correct.

When the lens is silvered on the surface of radius 60 cm, it will behave as a concave mirror of focal length f_M such that

$\frac{1}{f_M} = \sum \frac{1}{f} = \frac{1}{f_{lens}} + \frac{2}{R} + \frac{1}{f_{lens}} = \frac{1}{30} + \frac{2}{60} + \frac{1}{30} = \frac{6}{60} = \frac{1}{10}$ *means* $f_M = 10\,cm$

Hence the option b is correct.

When the image space is filled with a liquid of refractive index $\mu_2 = \frac{5}{3}$, the object space still being air ($\mu_1 = 1$), the second focal length of the lens is obtained by $\frac{5}{3f_2} = \frac{1.5 - 1}{+20} - \frac{1.5 - \frac{5}{3}}{-60}$

$\Rightarrow f_2 = +\,75\,cm$ Also the first focal length in this case is $f_1 = +\,45\,cm$ so the lens still behaves as convex lens and not a concave (diverging) lens. Hence option c is wrong.

Considering a different situation when air in object space and water $\left(\mu = \frac{4}{3}\right)$ in image space, the second focal length of lens then is $\frac{4}{3f_2} = \frac{1.5 - 1}{20} - \frac{1.5 - 4/3}{-60} = \frac{1}{36} \Rightarrow f_2 = +\,48\,cm$

Hence a beam of light incident parallel to the principal axis focuses 48 cm behind the lens. Hence option d is correct. **Ans: a, b, d**

32. A poorly conducting thick hollow cylinder is placed coaxially inside a long solenoid. If we consider a circle of radius r (a < r < b), the magnetic flux through this area shall be $\phi = \pi r^2 \beta t$

The induced emf therefore shall be $\varepsilon = -\frac{d\phi}{dt} = -\,\pi r^2 \beta$ Thus $|\varepsilon| = \pi r^2 \beta$ If R be the resistance offered to the circulating current then $\frac{1}{R} = \int_a^b \frac{hdr}{\rho \times 2\pi r} = \frac{h}{2\pi\rho} \ln\frac{b}{a}$ Thereby

$R = \frac{2\pi\rho}{h \times \ln\frac{b}{a}}$ Further the circulating current induced in the thick hollow poorly conducting cylinder is $i = \frac{\varepsilon}{R} = \int_a^b \pi r^2 \beta \frac{hdr}{\rho 2\pi r} = \frac{\beta h}{2\rho}\int_a^b rdr = \frac{\beta h}{4\rho}\left(b^2 - a^2\right)$

The time varying magnetic field parallel to the axis of the solenoid produces an electric field even outside the solenoid. The lines of force being circular with their centres lying on the axis of the solenoid so option d is wrong. **Ans: a, b, c**